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An Improvement of the Simple Model for Rotary Flow Cyclones

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A simplified model for the performance of a rotary flow cyclone has been reported by Ciliberti (1976) which has several inherent assumptions in it. A similar analysis is presented here that is somewhat more general, yet leading to expressions which are no more cumbersome and are in slightly better agreement with experimental observations.

DISCUSSION

The previous analysis of particle motion in the case of a rotary flow cyclone was based on the equation of drag force on a particle of a given size to the centrifugal force experienced by the particle. In the expression for the drag force, the inward radial gas velocity was neglected, giving rise to the equation

$$3\pi\mu d\left(\frac{dr}{dt}\right) = \frac{\rho_p \pi d^3}{6}\,(r\omega^2) \tag{1}$$

The determination of the angular velocity was based on an angular momentum balance about a cylindrical section of the core in which only the flux of momentum across the cylindrical area was accounted for, and the flux through the bottom and top were neglected. The following analysis attempts to include these factors.

A force balance on the particle shown in Figure 1 leads to the following expression:

$$\text{drag force} = \text{centrifugal force}$$

$$3\pi\mu d\left(\frac{dr}{dt} - V_r\right) = \frac{\pi d^3 \rho_p}{6}\,(r\omega^2) \tag{2}$$

This assumes Stokes Law drag forces, negligible acceleration in the radial direction, spherical, nonagglomerating particles, and that the tangential gas and particle velocities are equal. To solve this equation, V_r and ω must be determined.

The angular velocity may be obtained by looking at a section of the core which is assumed to be in solid body rotation. Figure 2 indicates the fluxes of angular momentum across this section's boundaries and leads to the following angular momentum balance:

$$\int_z^{z+\Delta z} \omega R_o \cdot R_o \cdot 2\pi R_o \rho V_r(R_o)\,dz + \int_o^{R_o} \omega r \cdot r \cdot 2\pi r \rho V_z dr \Big|_z = \int_o^{R_o} \omega r \cdot r \cdot 2\pi r \rho V_z dr \Big|_{z-\Delta z} \tag{3}$$

Performing the indicated integrations over r by assuming a flat V_z profile, and taking the limit as $\Delta z \to 0$, we obtain

$$\frac{d}{dz}(\omega V_z) = \frac{4V_r(R_o)\omega}{R_o} \tag{4}$$

The further assumption that the secondary gas flow enters the core uniformly leads to the expressions for V_z and $V_r(R_o)$:

$$V_r(R_o) = \frac{-Q_s}{2\pi R_o H} \tag{5}$$

$$V_z(z) = \frac{Q_p + \left(\frac{z}{H}\right) Q_s}{\pi R_o^2} \tag{6}$$

Substitution of these expressions into Equation (4) leads to this expression for $\omega(z)$:

$$\omega(z) = \omega(o)\left[1 + \left(\frac{z}{H}\right)\frac{Q_s}{Q_p}\right] \tag{7}$$

To determine the initial value of ω, some assumptions must be made about the design of the cyclone. It would be possible, for example, to evaluate ω if the value of S for the cyclone were known. Various values of S have

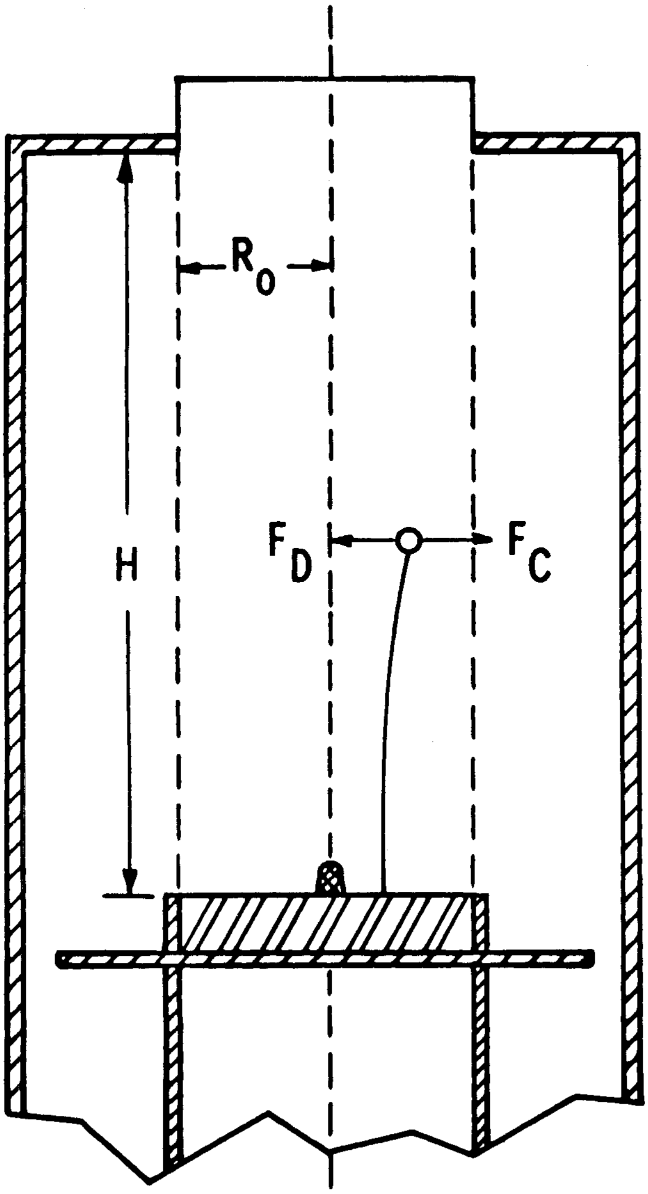

Fig. 1. Force diagram for particle in core of rotary flow cyclone.

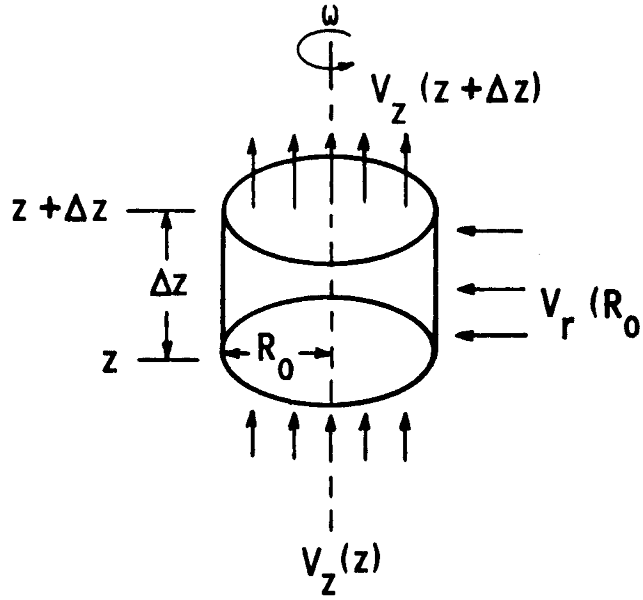

Fig. 2. Angular momentum balance for rotary flow cyclone.

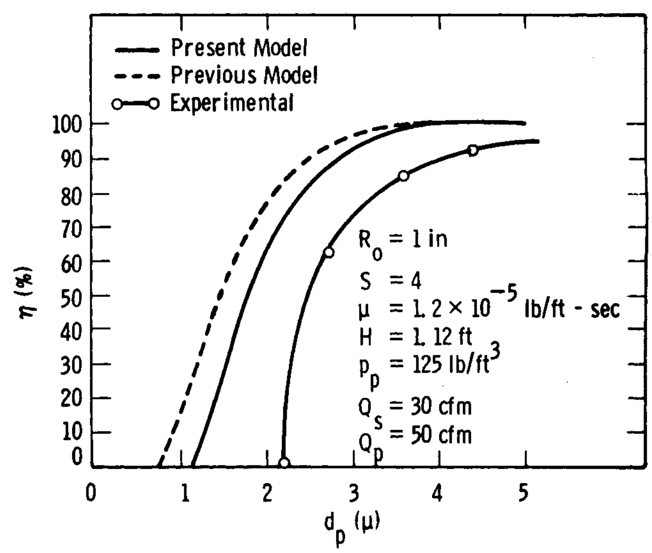

Fig. 3. Grade efficiency curves for 50 ft³/min unit.

been reported for high efficiency cyclones. Dalla Valla (1952) suggests a value of 3, while Friedlander et al. (1952) suggest 5. A value of 4 has been assumed here for rotary flow units. To use this information, consider the motion of the core section over the time increment dt:

$$dt = \frac{dZ}{V_Z} = \frac{d\theta}{\omega} \tag{8}$$

If the values of $V_Z(z)$ from Equation (6) and $\omega(z)$ from Equation (7) are substituted into Equation (8) and the resultant equation is integrated over the length of the cyclone, and through the angle of $2\pi S$, the resulting equation for the angular velocity is

$$\omega(z) = \frac{2Q_pS}{R_o^2H}\left[1+\left(\frac{z}{H}\right)\frac{Q_s}{Q_p}\right] \tag{9}$$

The determination of $V_r(r)$ for $r \leqq R_o$ can be accomplished through the use of the continuity equation

$$\frac{\partial V_z}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rV_r) = 0 \tag{10}$$

Since V_z has been established as a linear function of z, the Equation (10) can be written as

$$\frac{1}{r}\frac{d}{dr}(rV_r) = -\frac{Q_s}{\pi R_o^2H} \tag{11}$$

which, when integrated subject to the boundary condition of

$$V_r(R_o) = \frac{-Q_s}{2\pi R_oH} \tag{12}$$

leads to

$$V_r(r) = \left(\frac{-Q_s}{2\pi R_o^2H}\right)r; \quad r \leqq R_o \tag{13}$$

Returning to Equation (2) and assuming that the particle axial velocity is the same as the gas axial velocity so that

$$\frac{dr}{dt} = \frac{dz}{dt}\frac{dr}{dz} = V_z\frac{dr}{dz} \tag{14}$$

one obtains the following equation:

$$\frac{1}{r}\frac{dr}{dz} = \left[\frac{\rho_pd^2\omega^2(z)}{18\mu} - \frac{Q_s}{2\pi R_o^2H}\right]\frac{1}{V_z(z)} \tag{15}$$

By integration from an inlet position r to the core boundary R_o, where the particle is considered captured, and over the length H of the cyclone, the grade efficiency of the cyclone can be established as

$$\ln\left(\frac{R_o}{r}\right) = \frac{1}{2}\ln\left(\frac{1}{1-\eta}\right) = \left(\frac{\rho d^2}{18\mu}\right)(2\pi S)^2 \left[\frac{Q_p + \frac{1}{2}Q_s}{\pi R_o^2 H}\right] - \frac{1}{2}\ln\left(1 + \frac{Q_s}{Q_p}\right) \quad (16)$$

or

$$d_\eta = \left(\frac{3}{2\pi s}\right)\sqrt{\left(\frac{\mu}{\rho_p}\right)(\tau)\ln\left[\left(\frac{1}{1-\eta}\right)\left(1 + \frac{Q_s}{Q_p}\right)\right]} \quad (17)$$

where τ is the residence time in the cyclone:

$$\tau = \frac{\pi R_o^2 H}{Q_p + \frac{1}{2}Q_s} \quad (18)$$

It is interesting to note that the inclusion of the radial gas velocity term leads to the conclusion that the cyclone will only capture particles above a certain minimum size given by

$$d_o = \left(\frac{3}{2\pi s}\right)\sqrt{\left(\frac{\mu}{\rho_p}\right)(\tau)\ln\left[1 + \frac{Q_s}{Q_p}\right]} \quad (19)$$

This conclusion was previously arrived at by balancing the drag and maximum centrifugal force at the core boundary which yielded a conservative estimate of the minimum particle size. Figure 3 indicates the grade efficiencies determined by the previous method, the present method, and experimentally for a 50 ft^3/min unit. As the curve indicates, the inclusion of the radial gas velocity impedes particle collection, and the entire grade efficiency curve is shifted towards the experimental curve. The difference between the experimental results and the present model is most likely explained by the fact that the assumption of uniform flow of the secondary gas into the core is only an approximation. Turbulence, and its effects on particle reentrainment and capture, cannot be accounted for in this model, and this too may be an important cause for the discrepancy between experimental and calculated results.

A final observation to be made is that the model predicts different sized cyclones can maintain similar grade efficiencies for a given gas-particle system by maintaining

$$\frac{\sqrt{\tau}}{s} = \text{constant}$$

and

$$\frac{Q_s}{Q_p} = \text{constant}$$

NOTATION

d = particle diameter
H = length of cyclone body
Q_p = primary gas flow rate
Q_s = secondary gas flow rate
R_o = radius of rotational flow core
r = radius
S = number of revolutions of gas flow in core
t = time
V_z = gas axial velocity
V_r = gas radial velocity
z = axial position

Greek Letters

η = collection efficiency
μ = gas viscosity
ρ_p = particle density
τ = residence time in the core
ω = angular velocity

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Velocity Measurements in Two-Phase Bubble-Flow Regime with Laser-Doppler Anemometry

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Several analytical and experimental studies do exist in the area of two-phase flow involving a gas and a Newtonian liquid; in spite of these, it is still recognized that two-phase studies remain a complex area where the progress has been rather slow. Mahalingam and Valle (1972) and Mahalingam (1975) point out the existence of discrepancies between analytical and experimental results in two-phase, gas-liquid flow. In order to provide an improved interpretation of experimentally observed two-phase flow phenomena through analytical models, it is necessary a priori to develop novel experimental techniques to generate reliable data. The techniques described here provide for an accurate measurement of bubble rise velocity in a stagnant fluid and of the velocity of individual phase (and hence the slip velocity) in two-phase gas-liquid systems, in the bubble-flow regime. The laser-Doppler anemometer (LDA) is used in these measurements, thus confirming the potential of LDA systems in two-phase flow measurements.

The laser-Doppler anemometer has, over the last few years, rapidly evolved as a standard and absolute technique for measurement of velocities of single-phase systems such as liquids or gases. Doppler phenomenon is readily observed, where two laser beams are brought to a

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